LECTURE NO 18

Electrostatics

TOPIC COVERED

- Expression for electric field intensity due to a electric dipole at point P

... evident in the subsequent chapters.

Consider the dipole shown in Figure 4.20. The potential at point $P(r, \theta, \phi)$ is given by

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_o}\left[\frac{r_2 - r_1}{r_1 r_2}\right] \tag{4.77}$$

where r_1 and r_2 are the distances between P and $+Q$ and P and $-Q$, respectively. If $r \gg d$, $r_2 - r_1 \simeq d\cos\theta$, $r_2 r_1 \simeq r^2$, and eq. (4.77) becomes

$$V = \frac{Q}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2} \tag{4.78}$$

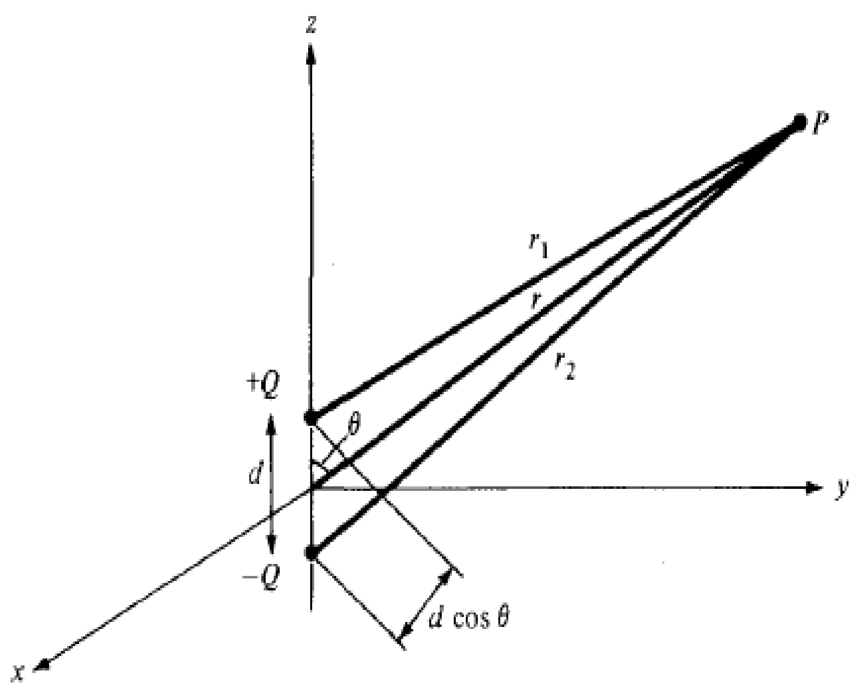

Figure 4.20 An electric dipole.

Since $d \cos\theta = \mathbf{d} \cdot \mathbf{a}_r$, where $\mathbf{d} = d\mathbf{a}_z$, if we define

$$\mathbf{p} = Q\mathbf{d} \tag{4.79}$$

as the *dipole moment*, eq. (4.78) may be written as

$$V = \frac{\mathbf{p} \cdot \mathbf{a}_r}{4\pi\varepsilon_0 r^2} \tag{4.80}$$

Note that the dipole moment $\mathbf{p}$ is directed from $-Q$ to $+Q$. If the dipole center is not at the origin but at $\mathbf{r}'$, eq. (4.80) becomes

$$V(\mathbf{r}) = \frac{\mathbf{p} \cdot (\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|^3} \tag{4.81}$$

The electric field due to the dipole with center at the origin, shown in Figure 4.20, can be obtained readily from eqs. (4.76) and (4.78) as

$$\begin{aligned} \mathbf{E} &= -\nabla V = -\left[\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta\right] \\ &= \frac{Qd\cos\theta}{2\pi\varepsilon_o r^3}\mathbf{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_o r^3}\mathbf{a}_\theta \end{aligned}$$

or

$$\mathbf{E} = \frac{p}{4\pi\varepsilon_o r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta) \tag{4.82}$$

where $p = |\mathbf{p}| = Qd$.